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Spontaneous Breaking of Lie and Current Algebras

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The anomalous properties of Nambu–Goldstone bosons, found by Miransky and others in the symmetry breaking induced by a chemical potential, are attributed to the SSB of Lie and current algebras. Ferromagnetism, antiferromagnetism, and their relativistic analogs are discussed as examples.[2]

KEY WORDS: Symmetry breaking; Nambu–Goldstone boson; color superconductivity; chemical potential; ferromganetism; Lorentz symmetry; current algebra.

1. INTRODUCTION AND SUMMARY

In general the number of the Nambu–Goldstone (NG) bosons associated with a spontaneous symmetry breaking (SSB) $G \to H$ is equal to the number of symmetry generators Q_i in the coset G/H. In the absence of a gauge field, their energy ω goes as a power k^γ of wave number. In a relativistic theory, $\gamma = 1$ necessarily unless Lorentz invariance is broken.

There are, however, exceptions to the above "theorem."[(1–5)] Recently one was found to occur in connection with color superconductivity in high density quark matter, where finite quark masses act like a chemical potential, and can trigger a kaon condensation. Two of the expected three NG modes coalesce into one, with $\gamma = 2$. I would like to give a dynamical explanation to the phenomenon.

I will first state the main result, which has a general validity. Suppose a symmetry generator (charge) Q develops a vacuum expectation value $\langle Q \rangle = C$. If two other charges Q_i, Q_j are such that their commutator $[Q_i, Q_j] = iQ$, then their corresponding zero modes Z_i, Z_j behave like

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[2] An early version was reported at a YITP workshop, Dec. 2001 (Kyoto University).

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canonical conjugates of each other: $[Z_i, Z_j] = iC$.[3] Hence they belong to the same dynamical degree of freedom, and the number of NG bosons is thereby reduced to one per each such pair. The dispersion law $\gamma = 2$ is obtained by a more detailed analysis.

In the breaking of space-time symmetries, not surprisingly, standard as well as non-standard properties of zero modes also appear.[4, 6–8] These cases are largely outside the scope of the present article, but an analog of ferromagnetism will be discussed at the end as an example.

The chemical potential μ is an intensive parameter conjugate to a conserved charge Q. It rearranges the spectrum and thus may change the vacuum. In the Lagrangian it appears in the form of a constant time component of a gauge field. Here I work with the corresponding Hamiltonian: $H = H_0 - \mu Q$.

2. A RELATIVISTIC $U(2)$ MODEL

In the example[2] of a two-component complex spin 0 field (kaon) $\Phi^{(a)}$, $a = 1, 2$, with $U(2)$ symmetry broken to a $U(1)$ by the $U(1)$ charge Q_0:

$$\begin{aligned}
L &= (\partial_0 + i\mu)\, \Phi^\dagger \cdot (\partial_0 - i\mu)\, \Phi - \nabla\Phi^\dagger \cdot \nabla\Phi - m^2 \Phi^\dagger \cdot \Phi - \lambda(\Phi^\dagger \cdot \Phi)^2, \\
H &= \Pi^\dagger \cdot \Pi + \nabla\Phi^\dagger \cdot \nabla\Phi + m^2 \Phi^\dagger \cdot \Phi - \mu Q_0 + \lambda(\Phi^\dagger \cdot \Phi)^2, \\
\Pi &= (\partial_0 + i\mu)\, \Phi^\dagger, \qquad \Pi^\dagger = (\partial_0 - i\mu)\, \Phi, \qquad Q_0 = -i(\Pi \cdot \Phi - \Phi^\dagger \cdot \Pi^\dagger)
\end{aligned} \tag{1}$$

First ignore the kinetic energy term. Q_0 acts like a magnetic field and splits particle and antiparticle for both components equally like a sea-saw. With increasing μ, an SSB occurs when the lower ones touch the ground, and stay thereafter as zero modes. So there are only two NG modes instead of the expected three. This can also be understood as follows.

The classical minimum of H corresponds to a stationary state $\partial_q H = \partial_p H = 0$. Make the real field decomposition:

$$\begin{aligned}
\Phi^{(1)} &= (x + iy)/\sqrt{2}, \qquad \Pi^{(1)} = (p_x + ip_y)/\sqrt{2}, \\
\Phi^{(2)} &= (u + iw)/\sqrt{2}, \qquad \Pi^{(2)} = (p_u + ip_w)/\sqrt{2}
\end{aligned} \tag{2}$$

Thus

$$\begin{aligned}
H &= (1/2)[(p_x^2 + p_y^2 + p_u^2 + p_w^2 + (m^2 + k^2)(x^2 + y^2 + u^2 + w^2)] \\
&\quad - \mu(xp_y - yp_x + up_w - wp_u) + \lambda(x^2 + y^2 + u^2 + w^2)^2/4 \\
&= (1/2)[(p_x + \mu y)^2 + (p_y - \mu x)^2 + (p_u + \mu w)^2 + (p_u - \mu u)^2 \\
&\quad + (m^2 - \mu^2)(x^2 + y^2 + u^2 + w^2)] + \lambda(x^2 + y^2 + u^2 + w^2)^2/4
\end{aligned} \tag{3}$$

[3] A similar phenomenon was observed in the context of string theory.[18]

The minima are given by

$$\begin{aligned}
&p_x+\mu y=0, \qquad p_y-\mu x=0, \qquad p_u+\mu w=0, \qquad p_w-\mu u=0,\\
&m^2x-\mu p_y=m^2y+\mu p_x=m^2u-\mu p_u=m^2w+\mu p_u\\
&\quad=-\lambda(x^2+y^2+u^2+w^2)\equiv-\lambda R^2, \qquad \text{or}\\
&\lambda R^2=\mu^2-m^2\ (>0\ \text{assumed})
\end{aligned} \tag{4}$$

Choose a minimum at $x=v$, $p_y=\mu v$, $v^2=(\mu^2-m^2)/\lambda>0$. The canonical momentum is nonzero since it is not just a time derivative. Make the shifts

$$x=v+\tilde{x}, \qquad p_y=\mu v+\tilde{p}_y \tag{5}$$

where the tilded quantities may be assumed to have no spatially constant components. Dropping the tilde for simplicity hereafter, the Hamiltonian is turned into

$$\begin{aligned}
H=&(1/2)[(p_x+\mu y)^2+(p_y-\mu x)^2+(p_u+\mu w)^2+(p_w-\mu u)^2\\
&+k^2(x^2+y^2+u^2+w^2)]+(\mu^2-m^2)\,x^2\\
&-\mu^4/4\lambda+\text{nonlinear terms}\\
=&(p_x^2+p_y^2+p_u^2+p_w^2)/2\\
&+(\mu^2+k^2)(x^2+y^2+u^2+w^2)/2+(\mu^2-m^2)\,x^2\\
&-\mu(xp_y-yp_x+up_w-wp_u)-\mu^4/4\lambda+\text{nonlinear terms}
\end{aligned} \tag{6}$$

and generate v-dependent NG terms Z_i in Q_i:

$$\begin{aligned}
&Q_i\rightarrow Q_i'=Q_i+Z_i,\\
&Z_0=Z_3=v(p_y+\mu x)+\mu v^2,\\
&Z_1=v(p_w+\mu u), \qquad Z_2=v(p_u-\mu w)
\end{aligned} \tag{7}$$

Then one finds

$$[Z_1, Z_2]=2iv^2\mu \tag{8}$$

consistent with the Lie algebra relation $[Q_1', Q_2']=2iQ_3'$. So Z_1 and Z_2 belonging to $\Phi^{(2)}$ are canonical conjugates of each other. Together they represent the same dynamical degree of freedom.

3. THE DISPERSION LAW

The dispersion law for small k may be found in the following general way. For this purpose it suffices to ignore nonlinear interaction terms as well as couplings between the NG and non-NG (Higgs) part of the collective modes, since the latter appear only through the kinetic term and thus of higher order in k. The collective part of the Hamiltonian (density), representing the NG and the Higgs part may be assumed to be of the quadratic form $H_{\text{col}} = A(p, q) + k^2 B(p, q)$ in their appropriate canonical coordinates p and q. (The k dependence can be nonisotropic.) It is also safe to assume $[A, B] \neq 0$ (unless $A = 0$), namely that the kinetic term does cause excitations. Then there are three possibilities:

(1) A depends on both p and q: $A = A(p, q)$,

(2) $A = A(p)$ (or equivalently $A = A(q)$),

(3) $A = 0$.

Case (1). At $k = 0$, H takes the form of a harmonic oscillator, so the frequency ω will go as const. $+ O(k^2)$ for small k. This is the massive (Higgs) mode.

Case (2). $A(p)$ (or $A(q)$) and $B(p, q)$ together essentially form a harmonic oscillator, so $\omega \sim k$. This is the regular case of the NG mode with phononlike dispersion law. In the above example it holds for the charges $Z_0 = Z_3$ coming from the $\Phi^{(1)}$ component.

Case (3). This applies to charges Z_1 and Z_2 coming from $\Phi^{(2)}$. If present, they would appear as conjugates and massive like the Case 1, which is a contradiction for zero modes. Hence $A = 0$, $B = B(p, q)$, and $\omega \sim k^2$ trivially.

These results are in conformity with the general theorem of Nielsen and Chada.[(1)]

4. FERRO- AND ANTIFERROMAGNETISM

Although the results derived above were motivated by symmetry breaking due to a chemical potential, it is clear that they should be valid generally, with or without a chemical potential, for relativistic as well as nonrelativistic examples, and even if the Hamiltonian is not expressed in the field theory language. The condensate or order parameter Q need not be one of the symmetry generators. Suppose a condensate forms for an operator O, $\langle O \rangle = C$. If a set of generators Q_i do not commute with O: $[Q_i, O] = O_i \neq 0$, the zero modes Z_i may be represented by the low-k

Fourier components corresponding to either of the sets O_i or Q_i, and an effective quadratic Hamiltonian can be constructed for them from the original equations of motion. Then any pair whose commutator yields Q leads to $[Z_i, Z_j] = iC$, so they are canonical conjugates, and the above theorems should apply.

Ferro- and antiferromagnetism are two examples of the above statement.[1] In the case of ferromagnetism, the symmetry operators are the total spin

$$S_i = \sum \sigma_i/2, \qquad i = 1 \cdots 3 \tag{9}$$

the order parameter is $\langle S_3 \rangle$, say, and the relevant dynamical variables are the small-k Fourier components $S_i(k)$. Applied to a ground state, these operators will generate low-lying states for which an effective quadratic Hamiltonian may be constructed.[9] The condensate S_3 is nonzero, so the zero modes S_1, S_2 are canonical conjugates, and form a single zero mode with a dispersion law $\gamma = 2$.

In antiferromagnetism, the symmetry generators are again S_i, but the order parameter is in the graded (staggered) sum

$$\begin{gathered} O_i = \sum_{\text{even}} \sigma_i/2 - \sum_{\text{odd}} \sigma_i/2, \\ \langle O_3 \rangle = C, \qquad \langle S_3 \rangle = 0 \end{gathered} \tag{10}$$

The S_i and O_i satisfy the $so(4)$ algebra

$$[S_i, S_j] = [O_i, O_j] = i\epsilon_{ijk} S_k, \qquad [S_i, O_j] = i\epsilon_{ijk} O_k \tag{11}$$

Applying the symmetry generators S_1, S_2 to O_3 one gets $-O_2, O_1$. So either the former pair or the latter may be taken as representative of the zero modes. But since $[S_1, S_2] = [O_1, O_2] = S_3$, $\langle S_3 \rangle = 0$, the two modes are independent, and the dispersion law should be a phononlike $\gamma = 1$. The relation $[S_1, O_2] = -[S_2, O_1] = iO_3$ also implies that the pairs (S_1, O_2) and $(-S_2, O_1)$ serve as canonical variables for the two modes in an effective Hamiltonian.

The relation between the above two alternative sets can be further illustrated by the case of the usual relativistic $O(4)$ sigma model with the field O_μ, $\mu = 0 \cdots 3$, $\langle O_0 \rangle = C$. The broken symmetry generators are $\{Q_{0i}\}$ which, applied to O_0, lead to the other set $\{O_i\}$: $[Q_{0i}, O_j] = i\delta_{ij} O_0$. Since $[O_i, O_j] = 0$, $[Q_{0i}, Q_{0j}] = -iQ_{ij}$, $\langle Q_{ij} \rangle = 0$, the different modes are independent, but Q_i and the zero mode part Z_i of O_{0i} are the canonical variables for mode i. In fact one knows from the sigma model Lagrangian that $Z_i = C\dot{O}_i$.

5. CHEMICAL POTENTIAL AS A THERMODYNAMIC VARIABLE

In thermodynamics and statistical mechanics the chemical potential μ is conjugate to a conserved quantity Q, to be related by

$$-\partial_\mu J = \langle Q \rangle \tag{12}$$

where J is the (grand canonical) free energy. The relation also holds in quantum mechanics for the ground state energy E_0 because its wave function ψ minimizes $\langle H \rangle$ with respect to arbitrary variations, and in particular, the variation of μ in $\psi(\mu)$. Naturally it is also expected to hold in quantum field theory when an SSB occurs. The vacuum is a variational minimum with respect to the order parameter v:

$$\partial_\mu E_0 = \langle \partial_\mu H \rangle + \partial_v \langle H \rangle \, dv/d\mu = \langle \partial_\mu H \rangle \tag{13}$$

This property of the chemical potential does not extend to excitations since their Hamiltonian has v dependence but is outside of the minimization procedure. It is also to be noted that the excitation quanta are not necessarily eigenstates of Q. (In the $U(2)$ example above, the modes in $\Phi^{(2)}$ are diagonal in Q, but those in $\Phi^{(1)}$ are not.)

At finite temperature, the condensate should be determined by self-consistently minimizing the free energy J. This involves thermodynamic sums over the collective modes, which will also modify (renormalize) the parameters through higher order loops. The thermodynamic relation, Eq. (12), is then expected to be recovered, and the charge carried by a collective mode of energy ω will be given by $\langle Q_\omega \rangle = -\partial_\mu \omega$.

In the $U(2)$ example,

$$J = \lambda v^4/4 + (m^2 - \mu^2)\, v^2/2 + F(v, T) \tag{14}$$

where F is the thermal free energy determined from the quadratic part of the Hamiltonian, Eq. (3), without setting $\lambda v^2 = \mu^2 - m^2$:

$$\begin{aligned} H = {} & (1/2)(p_x^2 + p_y^2 + p_u^2 + p_w^2) - \mu(p_x y - p_y x + p_u w - p_w u) \\ & + (k^2 + m^2 + \lambda v^2)(x^2 + y^2 + u^2 + w^2)/2 + \lambda v^2 x^2 \end{aligned} \tag{15}$$

plus corrections from loop effects. In actuality, however, these formal properties of J are spoiled by the divergent zero-point energies of the collective modes, which exist even at $T = 0$, if one wants to compare unbroken and broken phases.

The divergences can be removed if a normal product is taken for H corresponding to subtractions according to the non-SSB free Hamiltonian.

To see this, consider again the $U(2)$ model. The normal product form $:H:$ of the Hamiltonian H is given by $:H: = H - \langle H \rangle$, where $\langle H \rangle$ is taken with respect to free Hamiltonian with mass m. For the quadratic Hamiltonian H_{col} for the collective modes after SSB, this gives

$$\begin{aligned}\langle H_{\text{col}} \rangle &= \langle (p_x^2 + p_y^2 + p_u^2 + p_w^2)/2 \\ &\quad + (k^2 + m^2 + \lambda v^2)(x^2 + y^2 + u^2 + w^2)/2 + \lambda v^2 x^2 \rangle \\ &\sim 1/(2\pi^3) \int dk^3 [2(k^2+m^2)^{1/2} + 3\lambda v^2 / 2(k^2+m^2)^{1/2}] \end{aligned} \tag{16}$$

(The chemical potential term does not contribute: $\langle Q \rangle = 0$.) On the other hand, the true zero-point energy E_z is half the sum of the individual eigenvalues of H_{col}. For large k, their difference turns out to be

$$E_z - \langle H_{\text{col}} \rangle \sim -1/(2\pi^3) \int dk^3 (3/4)(\lambda v^2)^2 / k^3 \tag{17}$$

This may be regarded as a correction to the “bare” vacuum energy $\lambda v^4/4$, or to the bare coupling λ:

$$\delta\lambda \sim -3\lambda/(2\pi)^3 \int dk^3/k^3 \tag{18}$$

Indeed it is equal to the one-loop renormalization of λ. With this interpretation, then, the zero-point energy divergence is removed by normal ordering.

Nonrelativistic theories differ from relativistic ones in some respects. In the first place, a chemical potential acts like a constant mass parameter since there is no distinction between a scalar and the time component of a four-vector, nor does it split particle from antiparticle since the letter is absent. Secondly, the normal-ordering is sufficient to remove the zero-point energies completely. There are no renormalization effects.

6. BREAKING OF SPACE-TIME SYMMETRIES

The reduction of the number of zero modes also occur in the breaking of space-time symmetries. But can the Lorentz generators develop a condensate? Since they are non-commutative, do the zero modes obey the above theorem and hence their dispersion law are not photonlike?

First one notes that the orbital part of Lorentz generators depends on the coordinates explicitly. Its condensates are not homogeneous, so the zero modes of the type under discussion do not exist.[(4)] What about the spin part? If a condensate is possible, how will the spin-orbit coupling affect the zero modes?

To examine these questions, consider a relativistic analog of the ferromagnet in the vacuum. The spin density of a fermion field is the spatial part of the chiral current $j_{5\mu} = \bar{\psi}\gamma_5\gamma_\mu\psi = \psi^\dagger(\rho_1, \sigma_i)\,\psi$, and it is also a part of the angular momentum density M_{0ik} satisfying the $su(2)$ (current) algebra relations.[4] It is interesting to note that the relations among the spatial components of axial (σ_i) and vector ($\rho_1\sigma_i$) currents (the latter corresponding to imaginary Lorentz boosts), stand in direct correspondence to those for ferro- and antiferromagnetism. But for the moment consider only a ferromagnetic analog, and take the nonlinear interaction

$$L_{\text{int}} = -(g/2)\, j_{5\mu}\, j^{5\mu} \tag{19}$$

to see if a solution for a condensate can be found. Here $g > 0$ is assumed to make the interaction attractive in the spatial channels. One may regard the interaction as being mediated by an axial vector field.[5]

Thus assume

$$\langle j_{5z}\rangle = v \tag{20}$$

i.e., the vacuum is polarized in the z direction (in a certain Lorentz frame where the vacuum is defined in the standard manner), breaking Lorentz invariance including T and TCP. The free part of the Dirac equation reads[(10)]

$$(\gamma\cdot p - m + gv\gamma_5\gamma_3)\,\psi = 0 \tag{21}$$

with gv having the dimension of mass. It leads to the dispersion law

$$\begin{aligned}
(p_+^2 - m^2)(p_-^2 - m^2) &= 4g^2m^2v^2, \qquad p_{\pm,z} = p_z \pm gv,\\
\omega^2 &= \vec{p}^{\,2} + m^2 + g^2v^2 \pm 2gv(p_z^2 + m^2)^{1/2}\\
&= p_x^2 + p_y^2 + ((m^2 + k_z^2)^{1/2} \pm gv)^2
\end{aligned} \tag{22}$$

[4] It is not the algebra of current-field identity, where the spatial components are regarded independent and hence commuting among themsleves.

[5] In the real world it may be generalized to the neutral current weak interaction in the Standard Model and beyond, and may provide a dynamical basis for the work of Kostelecky and collaborators,[(10)] where various physical issues are discussed. A different model is given in ref. 11. For a possibility of spontaneous generation of magnetic field in 2+1 dimensions, see ref. 12.

The branch with the negative sign above has various peculiarities, such as spacelike states, $\omega < k$, and negative group velocity relative to momentum. Setting these problems aside, this model is being considered here for the purpose of examining the properties of collective modes.

The gap equation takes the form

$$1 = (g/4\pi^2)[\Lambda^2/2 - 2m^2 \ln(\Lambda^2/(m^2 + c_0 g^2 v^2)) + c_2 g^2 v^2],$$
$$\text{(a)}\quad gv \ll m, \quad c_2 > 0, \tag{23}$$
$$\text{(b)}\quad gv \gg m, \quad c_2 < 0$$

to the lowest order in expansion in g/m or m/g. The logarithmic divergence does not depend on g, i.e., a super-fine tuning of g is required.[6] Case (a) should be discarded. The value $c_2 > 0$ implies that the solution is unstable: the Higgs mass is tachyonic since $m_H^2 \sim -g\partial_g(\langle j \rangle / g)$, and there is also no critical coupling strength; an arbitrary small g would induce SSB, yielding an arbitrary large v.

The collective modes are the four components of the axial current satisfying current algebra relations. The Higgs mode $j_5 z$ and the two other spatial NG modes are related by $su(2)$, whereas the third one, the chiral charge, commutes with the former three, except for Schwinger terms, which would not influence the present discussion unless they were q-numbers that could acquire an expectation value. Actually, a new Schwinger term also emerges (see below). The properties of these modes are determined from the correlation loops $\langle T(j_{5\mu}(x)\, j_{5\nu}(y))\rangle$. It is difficult to see how the NG mass can be zero since the propagators in the loops are invariant only under the total angular momentum operation. Indeed the actual calculation shows it to be nonzero, of order g^2.

The effective Lagrangian for the collective current field $V_\mu = j_{5\mu}$ for low momenta takes the form of a broken gauge theory with a Chern–Simons term in the 3-dimensional subspace (x, y, t):

[6] There are ambiguities depending on how one computes divergent tensorial quantities. Observe that the condensate term does not break chirality, so when $m = 0$ it can be eliminated by a chiral gauge transformation $\exp(izgv\gamma_5)$, which would imply that the condensate should be proportional to m, in contradiction with Eq. (23). This line of argument, similar to removing of a chemical potential, is not followed here. (The condensate in the time component, $\langle j_{50} \rangle$, would be a bone fide chiral chemical potential.) The problem could also be related to the chiral anomaly, although no gauge field is assumed here. Actual calculations were done in Euclidean metric in order to maintain invariance in the (x, y, t) subspace.

$$
\begin{aligned}
L &= L_{\mu\nu} V^{\mu} V^{\nu}, \\
L_{\mu\nu} &= A g_{\mu\nu} k^2 + B k_{\mu} k_{\nu} \\
&\quad + C g_{\mu\nu} k_3^2 + D g_{3\mu} g_{3\nu} k^2 + E g_{3\mu} g_{3\nu} k_3^2 \\
&\quad + F (g_{3\mu} k_{\nu} k_3 + g_{3\nu} k_{\mu} k_3) \\
&\quad + i G \epsilon_{3\mu\nu\lambda} k^{\lambda} \\
&\quad - M^2 g_{\mu\nu} - \Delta M^2 g_{3\mu} g_{3\nu}
\end{aligned}
\tag{24}
$$

The Lorentz invariant terms A and B are $O(\ln \Lambda)$. The terms breaking Lorentz invariance are caused by gv but C, D, E, and F are dimensionless, whereas G is $O(gv)$ and ΔM^2 is $O(g^2 v^2)$. They are all finite. The chiral current conservation $k \cdot V = 0$ is intrinsically broken by the fermion mass (and the anomaly).

The Chern–Simons (CS) term G may be interpreted as a reflection of the $su(2)$ current algebra among the spatial components of V.[7] It splits the x and y components into massless and massive chiral modes, and would change the dispersion law to $\gamma = 2$ for small $|k| \ll G$, even if the ordinary mass were zero, in conformity with the theorem above. But the ordinary mass is actually nonzero due to the spin-orbit coupling inherent in a Lorentz SSB. Altogether there are one massless mode and three massive modes. From the Lie algebra point of view it may appear strange that the CS term also involves the time component V_0 which commutes with the spatial components. One might say that this was forced by the residual $(2+1)$ D invariance. But it furthermore turns out that the condensate term in the Dirac equation induces in the equal-time current commutators, in addition to the conventional one, a new Schwinger term of the form

$$
\begin{gathered}
[j_0(0), j_i(x)] = C \epsilon_{30ik} \partial^k \delta^3(x), \\
C \sim gv\Lambda
\end{gathered}
\tag{25}
$$

The details will be reported elsewhere.

Bjorken has argued that the photon can be generated as a result of a vector current condensate (Dirac's "aether") $\langle j_{\mu} \rangle \equiv A_{\mu}$.[13–17] Lorentz invariance is not broken because the breaking occurs only in the gauge potential. The three zero modes orthogonal to A match the three components of the vector potential necessary to describe the electromagnetic field. It is essential for this that the current be a conserved quantity. A constant potential can be eliminated by a gauge transformation,[10] but from the present viewpoint it should rather be construed as a chemical potential, in

[7] The effect of a similar term in electromagnetism is discussed in refs. 10 and 19.

which case it will change the definition of the vacuum and affect thermodynamics. This is similar to the observation made above[3] regarding the gauging away of spin condensate for massless fermion. In that case, however, the chiral current was not conserved, and the NG bosons were massive. The vector and axial vector condensates may be compared respectively to those in antiferro- and ferromagnetism. (For that matter, the Pauli electromagnetic current also stands in a similar relation to the axial current.) Furthermore, it appears that the current conservation, the current algebra relations, and the theorems established here about NG bosons are interrelated. To be more precise, one sees a logical linkage:

vector condensate $\sim$ *antiferromagnetism* $\rightarrow$ $\gamma = 1$ $\leftarrow$ *unbroken Lorentz* $\leftarrow$ *conserved charge* on the one hand, and

axial vector condensate $\sim$ *ferromagnetism* $\rightarrow$ $\gamma = 2$ $\leftarrow$ *massive NG and broken Lorentz* $\leftarrow$ *nonconserved charge* on the other.

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REFERENCES

1. H. B. Nielsen and S. Chadha, *Nucl. Phys. B* **105**:445 (1976).
2. V. A. Miransky and I. A. Shovkovy, *Phys. Rev. Lett.* **88**:11601 (2002); hep-ph/0108178.
3. T. Schaefer *et al.*, hep-ph/0108210.
4. I. Low and A. V. Manohar, hep-th/0110285.
5. F. Sannino, hep-ph/0211367.
6. C. J. Isham, A. Salam, and J. Streater, *Phys. Rev. D* **2**:685 (1979).
7. N. Nakanishi and I. Ojima, *Phys. Rev. Lett.* **43**:91 (1979).
8. T. Kugo, H. Terao, and S. Uehara, *Prog. Theoret. Phys. Suppl.* **85**:122 (1985).
9. H. Leutwyler, *Phys. Rev. D* **49**:3033 (1994), and references therein.
10. D. Collady and V. A. Kostelecky, *Phys. Rev. D* **55**:6760 (1997); **58**:116002 (1998).
11. V. A. Kostelecky and R. Lehnert, *Phys. Rev. D* **63**:065008 (2001).
12. D. Weslowski and Y. Hosotani, *Nucl. Phys. B* **354**:396 (1995).
13. P. A. M. Dirac, *Proc. Roy. Soc. Ser. A* **209**:291 (1951).
14. J. D. Bjorken, *Ann. Phys.* **24**:74 (1963); hep-th/0111196.
15. G. S. Guralnik, *Phys. Rev. B* **136**:1404, 1417 (1964).
16. Y. Nambu, *Prog. Theor. Phys. Extra Number* **190** (1968); *Proc. of the Second Meeting on CPT and Lorentz Symmetry*, A. Kostelecky, ed. (World Scientific, 2002), p. 1.
17. P. Krauss and E. T. Tomboulis, hep-th/0203221.
18. S. R. Das, C. Gomez, and S.-J. Ray, hep-th/0203164; S. R. Das and C. Gomez, hep-th/0206062.
19. S. M. Carrol, G. B. Fried, and R. Jackiw, *Phys. Rev. D* **41**:1231 (1990).